CBSE Class – XII Mathematics
Sample Paper-02

Time allowed: 3 hours (M M: 100)

General Instructions:

(i) All questions are compulsory.

(ii) This question paper contains 29 questions.

(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.

(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.

(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.

(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section A

1. Give an example of a relation which is symmetric but not reflexive and transitive.

Ans. Let A = {1, 2, 3, 4}

Let R={(1,2),(2,1)}

2. Is the measure of 5 seconds is scalar or vector?

Ans. Scalar.

3. What is the domain of $\sin^{-1} x$?

Ans. [-1,1]

4. Show that $\begin{vmatrix} \sin 10^\circ & -\cos 10^\circ \\ \sin 80^\circ & \cos 80^\circ \end{vmatrix} = 1$

Ans. $\sin 10 . \cos 80 + \cos 10 \sin 80$

$= \sin(10 + 90)$

$= [\because \sin A \cos B + \cos A . \sin B = \sin(A+B)]$

$= \sin 90$

$= 1$

Section B

5. Find the principal value of $\cot^{-1}(-\frac{1}{\sqrt{3}})$**.**

Ans. Let $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \theta$

$$\cot\theta = \frac{-1}{\sqrt{3}}$$

We know that $\theta \in (0, \pi)$

$$\cot\theta = \cot\left(\pi - \frac{\pi}{3}\right)$$

$$\theta = \frac{2\pi}{3}$$

There four p.v of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{2\pi}{3}$

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6. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ **and** $A^2 = I$ **. Find relation given by** $\alpha^2 = I$ **.**

Ans. $A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & 2\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}$$

ATQ. $\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\alpha^2 + \beta\gamma = 1$$

$$\alpha^2 + \beta\gamma - 1 = 0$$

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7. Verify Rolle's Theorem for the function f(x) = x^2 + 2x – 8, x ∈ [-4, 2]

Ans. The function $y = x^2 + 2x - 8, x \in [-4, 2]$

Continuous in [-4, 2] and differentiable in (-4, 2)

Also $f(-4) = f(2) = 0$

Hence all the condition of all Rolle's Theorem, is verified

Their exist a value C

Such that f' (c) = 0

f' (c) = 2c +2

0 = 2C+2

C = -1

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8. Find $\frac{dy}{dx}$ $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

Ans. $y = \tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$

$$y = \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$y = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

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9. $\int \frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}dx$

Ans. Put $\tan\sqrt{x} = t$

$$\sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}dx = dt$$

$$\frac{\sec\sqrt{x}}{\sqrt{x}}dx = 2dt$$

$$= 2\int t^4 dt = 2\frac{t^5}{5} + c$$

$$= \frac{2}{5}\tan^5\sqrt{x} + c$$

10. Verify that the function is a solution of the corresponding diff eq.

$x + y = \tan^{-1} y$; $y^2 y^1 + y^2 + 1 = 0$

Ans. $x + y = \tan^{-1} y$

$$1 + y^1 = \frac{1}{1+y^2} y^1$$

$$1 + y^2 + y^1 + y^1 y^2 = y^1$$

$$1 + y^2 + y^1 y^2 = 0$$

Hence proved.

11. Find the angle between vectors $\vec{a}$ *and* $\vec{b}$ *if* $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ $\quad \vec{a}.\vec{b} = \sqrt{6}$

Ans. $\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$

$$= \frac{\sqrt{6}}{(\sqrt{3}).(2)} = \frac{\sqrt{2}\times\sqrt{3}}{\sqrt{3}.2} = \frac{\sqrt{2}}{\sqrt{2}.\sqrt{2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \theta = \pi/4$$

12. A die thrown three times. Events A and B are defined as below.

A: 4 on the third throw

B: 6 on the first and 5 on the second throw.

Find the probability of A given that B has already occurred.

Ans. Total sample space = 216

$$A = \begin{bmatrix} (1,1,4) & (1,2,4)....(1,6,4) & (2,1,4) & (2,2,4)....(2,6,4) \\ (3,1,4) & (3,2,4)....(3,6,4) & (4,1,4) & (4,2,4)....(4,6,4) \\ (5,1,4) & (5,2,4)....(5,6,4) & (6,1,4) & (6,2,4)....(6,6,4) \end{bmatrix}$$

$$B = \{(6,5,1)\ (6,5,2)\ (6,5,3)\ (6,5,4)\ (6,5,5)\ (6,5,6)\}$$

$$A \cap B = \{6,5,4\}$$

$$P(B) = \frac{6}{216},\ P(A \cap B) = \frac{1}{216}$$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Section C

13. Solve: $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

Ans. $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$

$$3(2\tan^{-1}x) - 4(2\tan^{-1}x) + 2(2\tan^{-1}x) = \frac{\pi}{3}$$

$$2\tan^{-1}x = \frac{\pi}{3}$$

$$\tan^{-1}x = \frac{\pi}{6}$$

$$x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

14. Differentiate $\sin^{-1}\left[\frac{2^{x+1}}{1+4^x}\right]$

Ans. $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

$= \sin^{-1}\left[\frac{2^x.2}{1+(2^x)^2}\right]$

$Put\ 2^x = \tan\theta$

$= \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$y = 2.\tan^{-1} 2^x$

$\frac{dy}{dx} = 2.\frac{1}{1+(2^x)^2}.\frac{d}{dx}(2^x)$

$= \frac{2}{1+4^x}.2^x.\log 2$

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15. Use differentiation to approximate $(25)^{\frac{1}{3}}$

Ans. Let $x = 27,\ \Delta x = -2$ $y = x^{\frac{1}{3}}$

Let $x = 27,\ \Delta x = -2$

Then $\Delta y = (x+\Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}}$

$\Delta y = (25)^{\frac{1}{3}} - (27)^{\frac{1}{3}}$

$(25)^{\frac{1}{3}} = \Delta y + (27)^{\frac{1}{3}}$

$(25)^{\frac{1}{3}} = \Delta y + 3 --------- (1)$

$dy \sim \Delta y$

$dy = \left(\frac{dy}{dx}\right).\Delta x \quad [\because \Delta x = -2]$

$= \frac{1}{3}x^{\frac{-2}{3}}.(-2)$

$= -0.074 \qquad [x = 27]$

Put the value of dy in equation (1)

$(25)^{\frac{1}{3}} = 0.074 + 3$

$= 2.926$

16. Find a unit vector perpendicular to each of the vector

$\left(\vec{a}-\vec{b}\right)$ *and* $\left(\vec{a}+\vec{b}\right)$ *where* $\vec{a} = i + j + k$ *and* $\vec{b} = i + 2j + 3k.$

Ans. $\vec{a} = i + j + k$, $\vec{b} = i + 2j + 3k$

$\therefore \vec{a}+\vec{b} = 2i + 3j + 4k, \vec{a}-\vec{b} = -j - 2k$

$Vector \perp to\ \vec{a}+\vec{b}, \vec{a}-\vec{b}\ is\ \left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)$

$$\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right) = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix} = -2i + 4j - 2k = \vec{c}$$

$\therefore \left|\vec{c}\right| = 2\sqrt{6}$

Unit vector = $\dfrac{-2i + 4j - 2k}{2\sqrt{6}}$

17. If A and B are independent events such that P(AUB)=0.6, P(A)=0.2. Find P(B)

Ans. Since A and B are independent events $P(A\cap B) = P(A)P(B)$

$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0$

$\Rightarrow 0.6 = 0.2 + P(B) - .2P(B)$

$\Rightarrow P(B) = \dfrac{.4}{.8} = \dfrac{1}{2}$

18. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively of both try to solve the problem independently, find the probability that

(i) the problem is solved

(ii) Exactly one of them solves the problem.

Ans. E_1 : A solves the problem

E_2 : B solves the problem

P (E_1) = $\frac{1}{2}$ and P (E_2) = $\frac{1}{3}$

(i) P (the problem is solved)

= 1 – P (the problem is not solved)

$$= 1 - P\left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\right]$$

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

(ii) P Exactly one of their solves the problem

$$= \mathrm{P}(\mathrm{E}_1)\left(1-P(E_2)\right) + \mathrm{P}(\mathrm{E}_2)\left(1-P(E_1)\right)$$

$$= \frac{1}{2}\left(1-\frac{1}{3}\right) + \frac{1}{3}\left(1-\frac{1}{2}\right) = \frac{1}{2}$$

19. Solve $\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$

Ans. $\frac{dy}{dx} + \frac{2y}{3} = \frac{x}{\sqrt{y}}$

$$\sqrt{y}\frac{dy}{dx} + \frac{2y^{3/2}}{3} = x$$

$$Let\ z = y^{3/2} \Rightarrow \frac{dz}{dx} = \frac{3}{2}\sqrt{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{2}{3}\frac{dz}{dx} + \frac{2}{3}z = x$$

$$\Rightarrow \frac{dz}{dx} + z = \frac{3}{2}x$$

$$\therefore P = 1, Q = \frac{3}{2}x$$

$$\therefore I.F = e^{\int P dx} = e^x$$

Solution is:

$$ze^x = \int \frac{3}{2}xe^x dx$$

$$\Rightarrow y^{3/2}e^{x} = \frac{3}{2}xe^{x} - e^{x} + c$$

$$\Rightarrow y^{3/2} = \frac{3}{2}(x-1) + Ce^{-x}$$

20. If $A = \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}$ **and I is the identity matrix of order2, show that**

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Ans. $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}$

$$(I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + (\tan\alpha/2)\sin\alpha & -\sin\alpha + (\tan\alpha/2)\cos\alpha \\ \sin\alpha - (\tan\alpha/2)\cos\alpha & \cos\alpha + (\tan\alpha/2)\sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} (2\cos^2\alpha/2 - 1) + (\tan\alpha/2)(2\sin\alpha/2\cos\alpha/2) & -(2\sin\alpha/2\cos\alpha/2) + (\tan\alpha/2)(2\cos^2\alpha/2 - 1) \\ (2\sin\alpha/2\cos\alpha/2) - (\tan\alpha/2)(2\cos^2\alpha/2 - 1) & (2\cos^2\alpha/2 - 1) + (\tan\alpha/2)(2\sin\alpha/2\cos\alpha/2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix} = I + A$$

21. If $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots}}}$ *prove that* $(2y-1)\frac{dy}{dx} = \frac{1}{x}$.

Ans. $y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots}}}$

$$\Rightarrow y = \sqrt{\log x + y}$$

$$\Rightarrow y^2 = \log x + y$$

Differentiating both sides w.r.t x, we get

$$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \Rightarrow (2y-1)\frac{dy}{dx} = \frac{1}{x}$$

22. For any two vectors $\vec{a}$ *and* $\vec{b}$, *prove that* $|\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$.

Ans. $|\vec{a}+\vec{b}|^2 = (\vec{a}+\vec{b}).(\vec{a}+\vec{b})$

$= \vec{a}.\vec{a}+\vec{a}.\vec{b}+\vec{b}.\vec{a}+\vec{b}.\vec{b}$

$= |\vec{a}|^2 + 2\vec{a}.\vec{b}+|\vec{b}|^2 \le |\vec{a}|^2 + 2|\vec{a}.\vec{b}|+|\vec{b}|^2 \le |\vec{a}|^2 + 2|\vec{a}||\vec{b}|+|\vec{b}|^2$

$= (|\vec{a}|+|\vec{b}|)^2$

$\therefore |\vec{a}+\vec{b}| \le |\vec{a}|+|\vec{b}|$

23. Find the vector equation of the plane passing through the intersection of the planes $\vec{r}.(2i+2j-3k)=7, \vec{r}.(2i+5j+3k)=9$ *and the point* $(2,1,3)$.

Ans. $\overrightarrow{n_1} = 2i+2j-3k, d_1 = 7$

$\overrightarrow{n_2} = 2i+5j+3k, d_2 = 9$

Equation of plane:

$\vec{r}.(\overrightarrow{n_1}+\lambda\overrightarrow{n_2}) = d_1+\lambda d_2$

$\vec{r}.(2i+2j-3k+\lambda(2i+5j+3k)) = 7+9\lambda$

$Let\ \vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore x(2+2\lambda)+y(2+5\lambda)+z(-3+3\lambda) = 7+9\lambda$

$Putting\ (x,y,z)=(2,1,3)\ we\ get\ \lambda = \frac{10}{9}$

$Substituting\ the\ value\ of\ \lambda\ we\ get, \vec{r}.(38i+68j+3k) = 153$

Section D

24. Let A = R – {3} and B = R – {1}. Consider the function f **: A** $\to$ **B defined by** $f(x) = \left(\frac{x-2}{x-3}\right)$. **Is** f **one-one and onto? Justify your answer.**

Ans. A = R – {3} and B = R – {1} and $f(x) = \frac{x-2}{x-3}$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1 - 2}{x_1 - 3}$ and $f(x_2) = \frac{x_2 - 2}{x_2 - 3}$

Now, for $f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one function.

Now, $y = \frac{x-2}{x-3}$

$\Rightarrow y(x-3) = x-2$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1}$

$\therefore \ f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{3y - 2 - 2y + 2}{3y - 2 - 3y + 3} = y$

$\Rightarrow f(x) = y$

Therefore, f is an onto function.

25. Integrate $\int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

Ans. $I = \int \left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

$I = \int \left(\frac{1}{\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1}\right) dx$

$I = \int \frac{1 + \tan x}{\sqrt{\tan x}} dx$

$put\ \sqrt{\tan x} = t$

$\tan x = t^2$

$\sec^2 x\, dx = 2t\, dt$

$$dx = \frac{2t\, dt}{\sec^2 x}$$

$$= \frac{2t\, dt}{1+\tan^2 x}$$

$$= \frac{2t}{1+t^4}$$

$$= \int \frac{1+t^2}{t} \times \frac{2t}{1+t^4}\, dt$$

$$= 2\int \frac{t^2+1}{t^4+1} dt = 2\int \frac{t^2\left(1+\frac{1}{t^2}\right)}{t^2\left(1+\frac{1}{t^2}\right)} dt$$

$$2\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-2+2} dt$$

$$2\int \frac{1+1/t^2}{\left(t-\frac{1}{t}\right)^2+\left(\sqrt{2}\right)^2} dt$$

$put\ t-\frac{1}{t} = u$

$$\left(1-\frac{1}{t^2}\right) dt = du$$

$$= 2\int \frac{du}{(u)^2+\left(\sqrt{2}\right)^2}$$

$$= 2.\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c$$

$$= \sqrt{2}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}}+c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right)+c$$

$$= \sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+c$$

26. Find the shortest distance between the lines $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$
$\vec{r}=(2\hat{i}-\hat{j}-\hat{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$

Ans. $\vec{a}_1=\hat{i}+2\hat{j}+\hat{k}, \vec{b}_1=\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2=2\hat{i}-\hat{j}-\hat{k}, \vec{b}_1=2\hat{i}+\hat{j}+2\hat{k}$

$$d=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right).(\vec{b}_1\times\vec{b}_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

$\vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-2\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}$$

$=-3\hat{i}+3\hat{k}$

$$d=\left|\frac{(\hat{i}-3\hat{j}-2\hat{k}).(-3\hat{i}+3\hat{k})}{\left|-3\hat{i}+3\hat{k}\right|}\right|$$

$$=\left|\frac{-3-6}{\sqrt{9+9}}\right|=\left|\frac{9}{3\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$$

27. Using integration find the area of the region

$\{(x,y):x^2+y^2\leqslant 2ax, y^2\geqslant ax, x,y\geqslant 0\}$.

Ans. Given: $x^2+y^2\le 2ax, y^2\ge ax, x,y\ge 0$

$\Rightarrow x^2 + y \ -2ax \leq 0,\ y^2 \geq ax, x, y \geq 0$

$\Rightarrow x^2 + y^2 - 2ax + a^2 - a^2 \leq 0,\ y^2 \geq ax, x, y \geq 0$

$\Rightarrow (x-a)^2 + y^2 \leq a^2,\ y^2 \geq ax, x, y \geq 0$

To find the points of intersection of the circle $\left[(x-a)^2 + y^2 = a^2\right]$ and the parabola $\left[y^2 = ax\right]$, we will substitute $y^2 = ax$ in $(x-a)^2 + y^2 = a^2$.

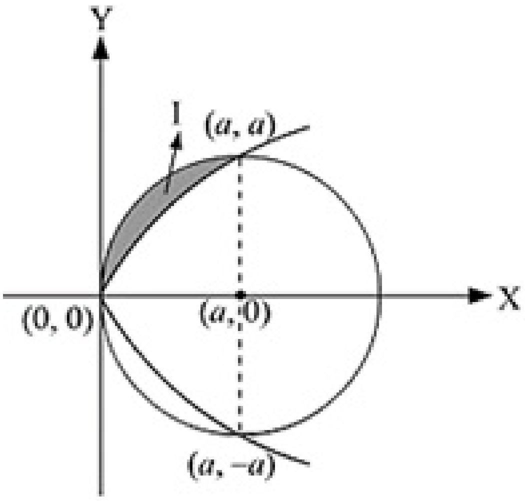
$(x-a)^2 + ax = a^2$

$\Rightarrow x^2 + a - 2ax + ax = a^2$

$\Rightarrow x(x-a) = 0$

$\Rightarrow x = 0, a$

Therefore, the points of intersection are (0, 0), (*a*, *a*) and (*a*, −*a*).



Now,

Area of the shaded region= *I*

Area of *I* from *x*=0 to *x*=*a*

$$= \left[\int_0^a \left(\sqrt{a^2 - (x-a)^2}\right)dx - \int_0^a \sqrt{ax}\,dx\right]$$

Let *x*−*a*=*t* for the first part of the integral $= \int_0^a \left(\sqrt{a^2 - (x-a)^2}\right)dx.$

$\Rightarrow dx = dt$

$$\therefore A_I = \int_{-a}^{0} \sqrt{a^2 - t^2}\,dt - 2\frac{\sqrt{a}}{3} \left| x^{\frac{3}{2}} \right|_0^a$$

$$= \left| \frac{t}{2}\sqrt{a^2 - t^2} + \frac{1}{2}a^2 \sin^{-1}\frac{t}{a} \right|_{-a}^{0} - \frac{2a^2}{3}$$

$$= 0 - \left(-\frac{\pi a^2}{4} \right) - \frac{2a^2}{3}$$

$$A_1 = \left(\frac{\pi}{4} - \frac{2}{3} \right) a^2$$

$\therefore$ Area of the shaded region $= \left(\frac{\pi}{4} - \frac{2}{3} \right) a^2$ square units

28. The sum of three numbers is 6. If we multiply the third number by 3 and add second number to it we get 11. By adding first and third numbers we get double of the second number. Represent the following information mathematically and solve using matrices.

Ans. Suppose the factory produces x units of machine A and y units of machine B.

Then, Profit Z= 10,500x + 9000y

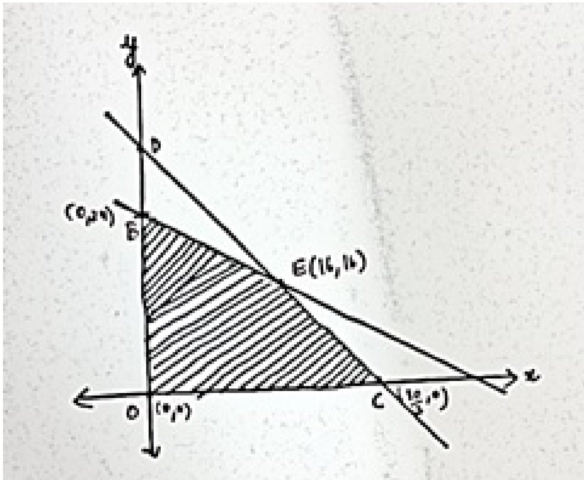
The mathematical formulation of the problem is as follows:

Max Z= 10,500x+9000y

s.t $10x+20y \leq 480$, $x+2y \leq 48$ (metal constraint)

$15x+10y \leq 400$, $3x+2y \leq 80$ (painting constraint)

$x \geq 0$, $y \geq 0$



We graph the above inequalities. The feasible region is as shown in the figure. We observe the feasible region is bounded and the corner points are O,B,E and C. The co-ordinates of the corner points are (0,0), (0,24),(16,16),(80/3,0).

Corner Point	Z=10,500x + 9000y

(0,0)	0
(0,24)	2,16,000
(16,16)	**3,12,000**
(80/3,0)	2,80,000

Thus profit is maximized by producing 16 units each of machine A and B.

29. A factory manufactures two types of machines A and B. Each type is made of certain metal. The factory has only 480kgs of this metal available in a day. To manufacture machine A, 10 kgs of metal is required and 20kgsis required for B. Machine A and B require 15 and 10minutes to be painted. Painting department can use only 400 minutes in a day. The factory earns profit of 10,500 on machine A and 9000 on machine B. State as a linear programming problem and maximizes the profit.

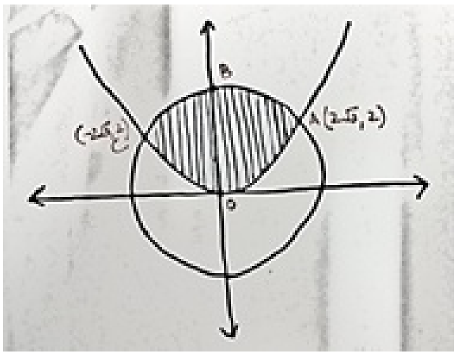
Ans. The point of intersection of the two curves:

$$x^2+y^2=16, x^2=6y$$

$$\Rightarrow y^2+6y-16=0$$

$$\Rightarrow y=2, -8$$

Rejecting y=-8, we get $x=\pm 2\sqrt{3}$.



Shaded area= Required area=Ar(OAB)+Ar(OBC)=2 Ar(OAB)

$$Area=2\int_0^{2\sqrt{3}}(y_1-y_2)dx=2\int_0^{2\sqrt{3}}\left(\sqrt{16-x^2}-\frac{x^2}{6}\right)dx$$

$$=2\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)-\frac{1}{6}\left(\frac{x^3}{3}\right)\Big|_0^{2\sqrt{3}}\right]$$

$$=2\left[2\sqrt{3}+\frac{8\pi}{3}-\frac{4\sqrt{3}}{3}\right]=\frac{4}{3}[4\pi+\sqrt{3}]$$